AAB University

Faculty of Computer Sciences

Introduction to Digital Technologies and Circuits

Week 2:

Number Systems and Conversions

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Last Time

- *An Introduction To Computer Science*
- *Fundamental Concepts in Computer Science*
- *Algorithms*
- *Digital Systems*
- *Binary Logic*
- *Information*

Today ☺

- Number Systems and Conversions:
  - Convert 53_{10} to binary $(?)_2$
  - Convert 0.625_{10} to binary $(?)_2$
  - Convert 0.7_{10} to binary $(?)_2$
  - Convert 53.7_{10} to binary $(?)_2$
  - Convert 1001111_2 to decimal $(?)_{10}$
  - Convert 2014_{10} to octal $(?)_8$
  - Convert 34.5_8 to decimal $(?)_{10}$
  - Convert 2014_{10} to hexadecimal $(?)_{16}$
  - Convert ABC_{16} to decimal $(?)_{10}$
  - **Convert 231.3_4 to base $(?)_7$**

Number Systems

- Number Systems?!

- Not weighted Number Systems
  - Roman: L, C, D, M, …

- Weighted Number Systems
  - Decimal
  - Binary
  - Octal
  - Hexadecimal
  - ….

$$N = \sum_{i=1}^{m} X_i \cdot B^{m-i} + \sum_{j=1}^{n} Y_j \cdot B^{-j}$$

$$X_i, Y_j = \{0, 1, \ldots, (B-1)\}$$

$B - Base$

Number Systems and Conversions

- When we write decimal (base 10) numbers, we use a positional notation; each digit is multiplied by an appropriate power of 10 depending on its position in the number. For Example:

$$(953.78)_{10} = 9\times10^{2} + 5\times10^{1} + 3\times10^{0} + 7\times10^{-1} + 8\times10^{-2}$$

- Similarly, for binary (base 2) numbers, each binary digit is multiplied by the appropriate power of 2:

$$(1011.11)_{2} = 1\times2^{3} + 0\times2^{2} + 1\times2^{1} + 1\times2^{0} + 1\times2^{-1} + 1\times2^{-2}$$

$$= 8 + 0 + 2 + 1 + 1/2 + 1/4$$

$$= 11 + 0.75 = (11.75)_{10}$$

Number Systems and Conversions

- Any positive integer B (B > 1) can be chosen as the base of a number system

$$N = (a_4\, a_3\, a_2\, a_1\, a_0 \cdot a_{-1} a_{-2}\, a_{-3})_B$$

$$= a_4 \times B^4 + a_3 \times B^3 + a_2 \times B^2 + a_1 \times B^1 + a_0 \times B^0$$

$$+ a_{-1} \times B^{-1} + a_{-2} \times B^{-2} + a_{-3} \times B^{-3}$$

Number Systems and Conversions

- Convert 34.7_8 to decimal $(?)_{10}$

Number Systems and Conversions

- Convert $\mathbf{ABC}_{16}$ to decimal $(\mathbf{?})_{10}$

Number Systems and Conversions

- Examples:
  - Convert 53_{10} to binary $(?)_2$
  - Convert 0.625_{10} to binary $(?)_2$
  - Convert 0.7_{10} to binary $(?)_2$
  - Convert 53.7_{10} to binary $(?)_2$
  - Convert 1001111_2 to decimal $(?)_{10}$ √
  - Convert 2014_{10} to octal $(?)_8$
  - Convert 34.5_8 to decimal $(?)_{10}$ √
  - Convert 2014_{10} to hexadecimal $(?)_{16}$
  - Convert ABC_{16} to decimal $(?)_{10}$ √
  - **Convert 231.3_4 to base $(?)_7$**

Number Systems and Conversions

- Convert 53_{10} to binary $(?)_2$

$$
\begin{array}{rll}
2 \,/\!\underline{53} & & \\
2 \,/\!\underline{26} & \text{rem.} = 1 = a_0 & \\
2 \,/\!\underline{13} & \text{rem.} = 0 = a_1 & \\
2 \,/\!\underline{6} & \text{rem.} = 1 = a_2 & \\
2 \,/\!\underline{3} & \text{rem.} = 0 = a_3 & \\
2 \,/\!\underline{1} & \text{rem.} = 1 = a_4 & \\
0 & \text{rem.} = 1 = a_5 &
\end{array}
$$

$(53)_{10} = (110101)_2$

Number Systems and Conversions

- Convert 0.625_{10} to binary $(?)_2$

.625	.250	.500
× 2	× 2	× 2
1.250	0.500	1.000
$(a_{-1} = 1)$	$(a_{-2} = 0)$	$(a_{-3} = 1)$

$(0.625)_{10} = 0.101_2$

Number Systems and Conversions

- Convert 0.7_{10} to binary $(?)_2$

Number Systems and Conversions

- Convert 53.625_{10} to binary $(?)_2$

$$(53.625)_{10} = (53 + 0.625)_{10}$$

$$(53)_{10} = (110101)_2 \quad \& \quad (0.625)_{10} = (0.101)_2$$

$$(53.625)_{10} = (110101)_2 + (0.101)_2$$

$$= (110101.101)_2$$

Number Systems and Conversions

- Examples:
  - Convert 53_{10} to binary $(?)_2$ √
  - Convert 0.625_{10} to binary $(?)_2$ √
  - Convert 0.7_{10} to binary $(?)_2$ √
  - Convert 53.7_{10} to binary $(?)_2$ √
  - Convert 1001111_2 to decimal $(?)_{10}$ √
  - Convert 2014_{10} to octal $(?)_8$ √
  - Convert 34.5_8 to decimal $(?)_{10}$ √
  - Convert 2014_{10} to hexadecimal $(?)_{16}$ √
  - Convert ABC_{16} to decimal $(?)_{10}$ √
  - **Convert 231.3_4 to base $(?)_7$** √

Number Systems and Conversions

- Direct conversions between number systems:

$$B = b^k \quad , k = 1,2,\ldots$$

- Examples:
  - Convert 10111011_2 to octal $(?)_8$
  - Convert 10111011_2 to hexadecimal $(?)_{16}$
  - Convert $A5.C_{16}$ to binary $(?)_2$
  - Convert $ABE.B_{16}$ to octal $(?)_8$
  - Convert 53.7_8 to hexadecimal $(?)_{16}$
  - **Convert 220012112021.102_3 to base $(?)_9$**

Number Systems and Conversions

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$B = b^k$$

Number Systems and Conversions

- Convert 10111011_2 to octal $(?)_8$

Number Systems and Conversions

- Convert 10111011_2 to hexadecimal $(?)_{16}$

Number Systems and Conversions

- Convert **ABE.B$_{16}$** to octal **(?)$_8$**

(ABE.B)$_{16}$ = (?)$_2$

(?)$_2$ = (?)$_8$

Number Systems and Conversions

- Convert 2014_{10} to octal $(?)_8$

$(2014)_{10} = (?)_2$

$(?)_2 = (?)_8$

OR

?

Number Systems and Conversions

- Convert 2014_{10} to hexadecimal $(?)_{16}$

$(2014)_{10} = (?)_{2}$

$(?)_{2} = (?)_{16}$

OR

?

Number Systems and Conversions

- More Examples ☺

- Convert 231.3_4 to binary $(?)_7$

?

- Convert 220012112021.102_3 to binary $(?)_9$

?

- Questions?!

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