AAB University
Faculty of Computer Sciences

Introduction to Digital Technologies and Circuits

Week 4:

Boolean Algebra and Logic Functions

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Last Time

- Binary Arithmetic
  - Addition
  - Subtraction
  - Multiply
  - Divide
- Octal Arithmetic
- Hexadecimal Arithmetic
- Binary Codes
  - Weighted Codes
  - Non Weighted Codes

Today

- Boolean Algebra
- Logic (Boolean) Functions
- Representations of Boolean Functions
  - Switching Circuits
  - Truth Tables
  - Timing Diagrams
  - Venn Diagrams
  - K-Diagrams

Boolean Algebra

- Let **B={0, 1}**,
  we can define three operations: **+** , **·** and **’** on **B** as follows:

+	1	0
1	1	1
0	1	0

·	1	0
1	1	0
0	0	0

	′
1	0
0	1

In other words:

$$0+0=0$$
$$0+1=1$$
$$1+0=1$$
$$1+1=1$$

$$0*0=0$$
$$0*1=0$$
$$1*0=0$$
$$1*1=1$$

$$\overline{0}=1$$
$$\overline{1}=0$$

- The operations **+** , **·** and **’** on **B={0, 1}** are called *Boolean sum* (the logical **OR**), *Boolean product* (the logical **AND**) and *Boolean Complementation* (the logical **NOT**)!

Boolean Algebra

- Definition: The set **B={0, 1}** together with Boolean **+** , **·** and **’** is called a **Boolean Algebra**.

- A **Boolean expression** is a sequence of zeros, ones, and *literals* separated by Boolean operators.

- A literal is a primed (negated) or unprimed **variable** name. For our purposes, all variable names will be a single alphabetic character (Ex. A, B, C, X, Y, …)

- A **Boolean function** is a specific Boolean expression; we will generally give Boolean functions the name “F” with a possible subscript. For example, consider the following Boolean: **$F_0 = AB+C$**

Postulates and Theorems

- For any given algebra system, there are some initial assumptions, or *postulates*, that the system follows. We can deduce additional rules, theorems, and other properties of the system from this basic set of postulates.

- Postulates:

$$0+0=0 \qquad 0*0=0 \qquad \overline{0}=1$$
$$0+1=1 \qquad 0*1=0 \qquad \overline{1}=0$$
$$1+0=1 \qquad 1*0=0$$
$$1+1=1 \qquad 1*1=1$$

- Theorems:

$$A+0=A \qquad A*0=0 \qquad \overline{\overline{A}}=A$$
$$A+1=1 \qquad A*1=A$$
$$A+A=A \qquad A*A=A$$
$$1+\overline{A}=1 \qquad A*\overline{A}=0$$

Postulates and Theorems

- We can prove all other theorems in Boolean algebra using these postulates, but we will not go into the formal proofs of these theorems, however, it is a good idea to familiarize our self with some important theorems in Boolean algebra. A sampling include:

(A + B)’ = A’ • B’ *or* $\overline{A+B} = \overline{A} \cdot \overline{B}$ *DeMorgan’s Theorems*

(A • B)’ = A’ + B’ $\overline{A \cdot B} = \overline{A} + \overline{B}$

A + A•B = A

A •(A + B) = A

A + A’B = A+B

A’ • (A + B’) = A’B’

AB + AB’ = A

(A’+B’) • (A’ + B) = A’

....

Dual and Inverse Functions

- For every Boolean (**Logic**) **Function**:

a) $F = A \cdot B + C$ b) $F = A \cdot \overline{B} + \overline{A} \cdot B$ c) $F = A \cdot B + C \cdot (\overline{A} + B)$

- If in *Logic Function* $\begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow 0 \\ + \rightarrow \cdot \\ \cdot \rightarrow + \end{matrix}$ the result is *Dual Function*:

a) $F_d = (A + B) \cdot C$ b) $F_d = (A + \overline{B}) \cdot (\overline{A} + B)$ c) $F_d = (A + B) \cdot [C + (\overline{A} \cdot B)]$

- If in *Dual Function* $\begin{matrix} A \rightarrow \overline{A} \\ \overline{A} \rightarrow A \end{matrix}$ the result is *Inverse Function*:

a) $\overline{F} = (\overline{A} + \overline{B}) \cdot \overline{C}$ b) $\overline{F} = (\overline{A} + B) \cdot (A + \overline{B})$ c) $\overline{F} = (\overline{A} + \overline{B}) \cdot [\overline{C} + (A \cdot \overline{B})]$

Representations of Boolean Functions

I. Switching Circuits

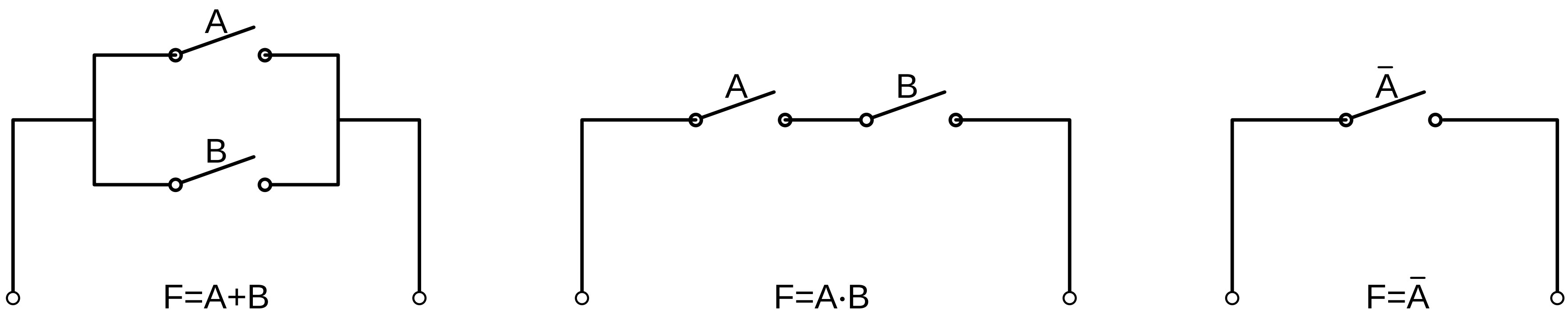
II. Truth Tables

III. Timing Diagrams

IV. Venn Diagrams

V. K-Diagrams

- Representations of Boolean Functions with Switching Circuits:



I. Switching Circuits

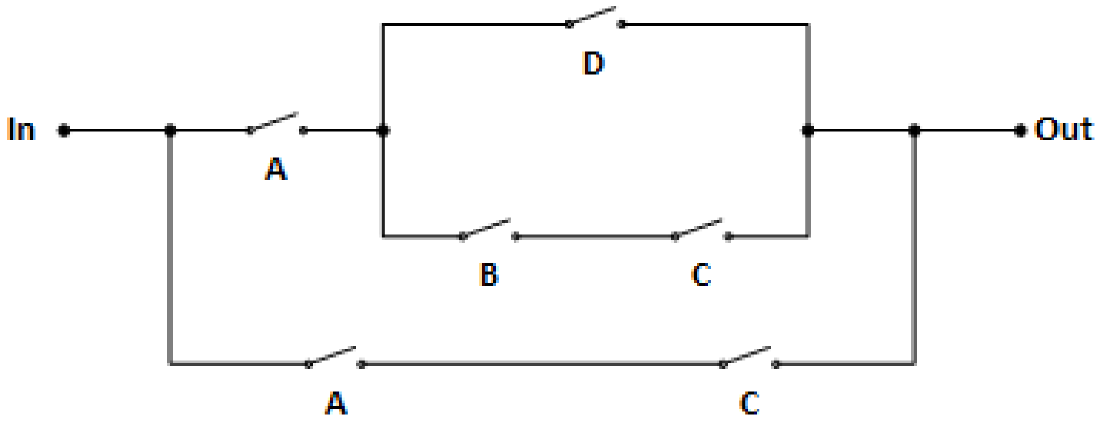
- Example 1:
  - *Draw the circuits for the following functions:*

$$a)\quad F_1 = A\cdot B + C$$

$$b)\quad F_2 = A\cdot \overline{B} + \overline{A}\cdot B$$

$$c)\quad F_3 = A\cdot B + C\cdot(\overline{A} + B)$$

- **Example 2**:
  - *For the following circuit, find the output and design a simpler circuit that has the same output!*



F = ?

II. Truth (Combinational) Tables

- Representations of Boolean Functions with Combinational Tables:

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

$$F = A + B$$

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

$$F = A \cdot B$$

A	F
0	1
1	0

$$F = \overline{A}$$

- **Minterms** and **Maxterms**

- The sum of minterms functions form:

$$F = \sum_{i=0}^{2^n-1} F_i \cdot m_i$$

- The product of Maxterms functions form:

$$F = \prod_{i=0}^{2^n-1} (F_i + M_i)$$

- Example 3:
  - *Draw the Truth (Combinational) Tables for the following functions:*

$$a)\quad F_1 = A\cdot B + C$$

$$b)\quad F_2 = A\cdot \overline{B} + \overline{A}\cdot B$$

$$c)\quad F_3 = A\cdot B + C\cdot(\overline{A} + B)$$

- Example 4:
  - *For the given Truth Tables, find the logic functions!*

a)

i	A	B	C	F_1
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

F_1 (A, B, C) = ?

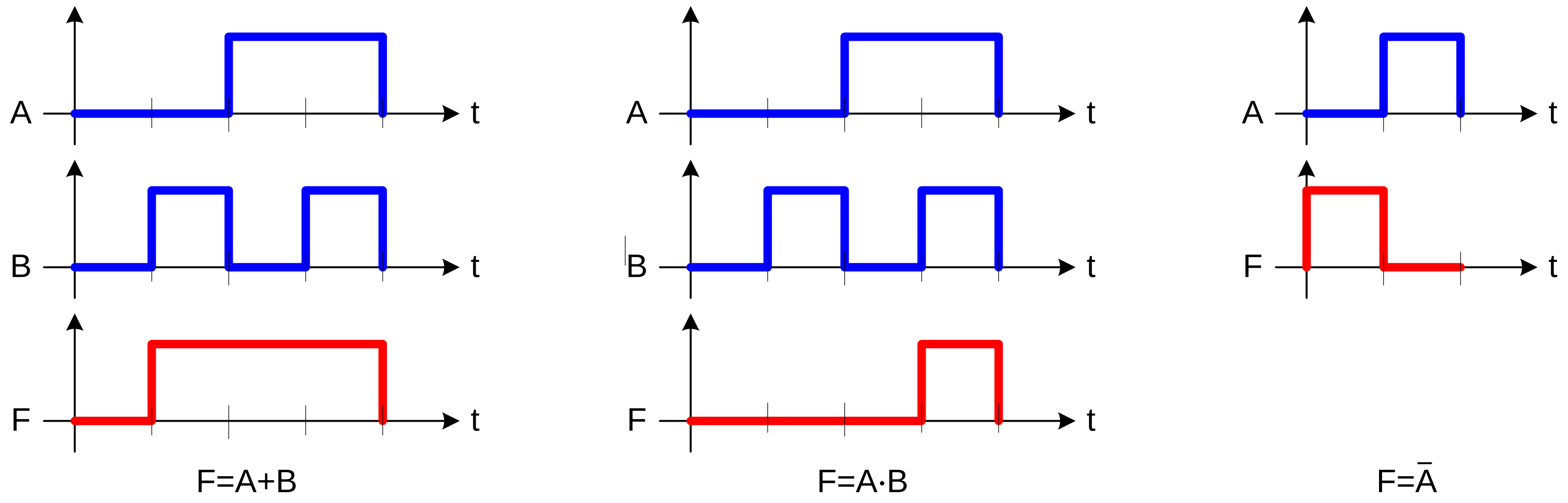
b)

i	F_2
0	0
1	0
2	1
3	1
4	1
5	1
6	0
7	0

F_2 = ?

III. Timing Diagrams

- Representations of Boolean Functions with Timing Diagrams:



III. Timing Diagrams

- Example 5:
  - *Draw the Timing Diagrams for the following functions:*

$$a)\quad F_1 = A \cdot B + C$$

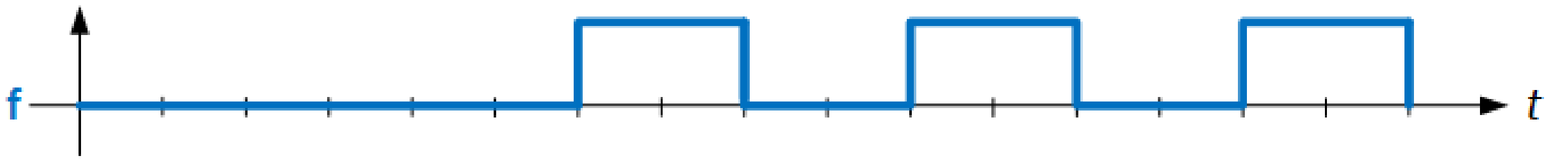
$$b)\quad F_2 = A \cdot \overline{B} + \overline{A} \cdot B$$

$$c)\quad F_3 = A \cdot B + C \cdot (\overline{A} + B)$$

- Example 6:
  - *For the given Timing Diagram:*

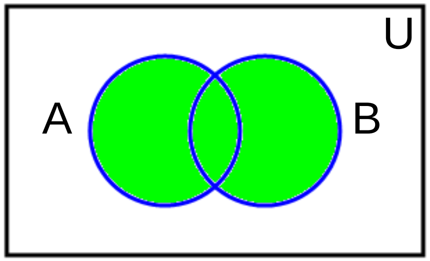
    a) Find the logic function and

    b) Simplify the function and draw the minimized circuit

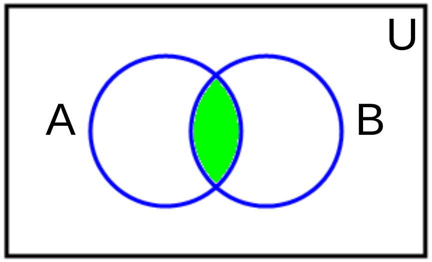


IV. Venn Diagrams

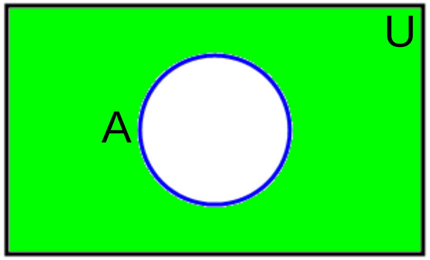
- Representations of Boolean Functions with Venn Diagrams:



F=A+B



F=A·B



F=$\bar{A}$

IV. Venn Diagrams

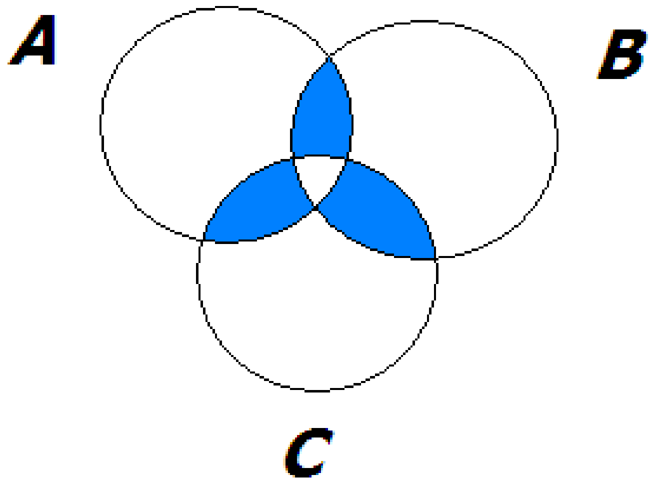
- Example 7:
  - *Draw the Venn Diagrams for the following functions:*

$$a)\quad F_1 = A\cdot B + C$$

$$b)\quad F_2 = A\cdot \overline{B} + \overline{A}\cdot B$$

$$c)\quad F_3 = A\cdot B + C\cdot(\overline{A} + B)$$

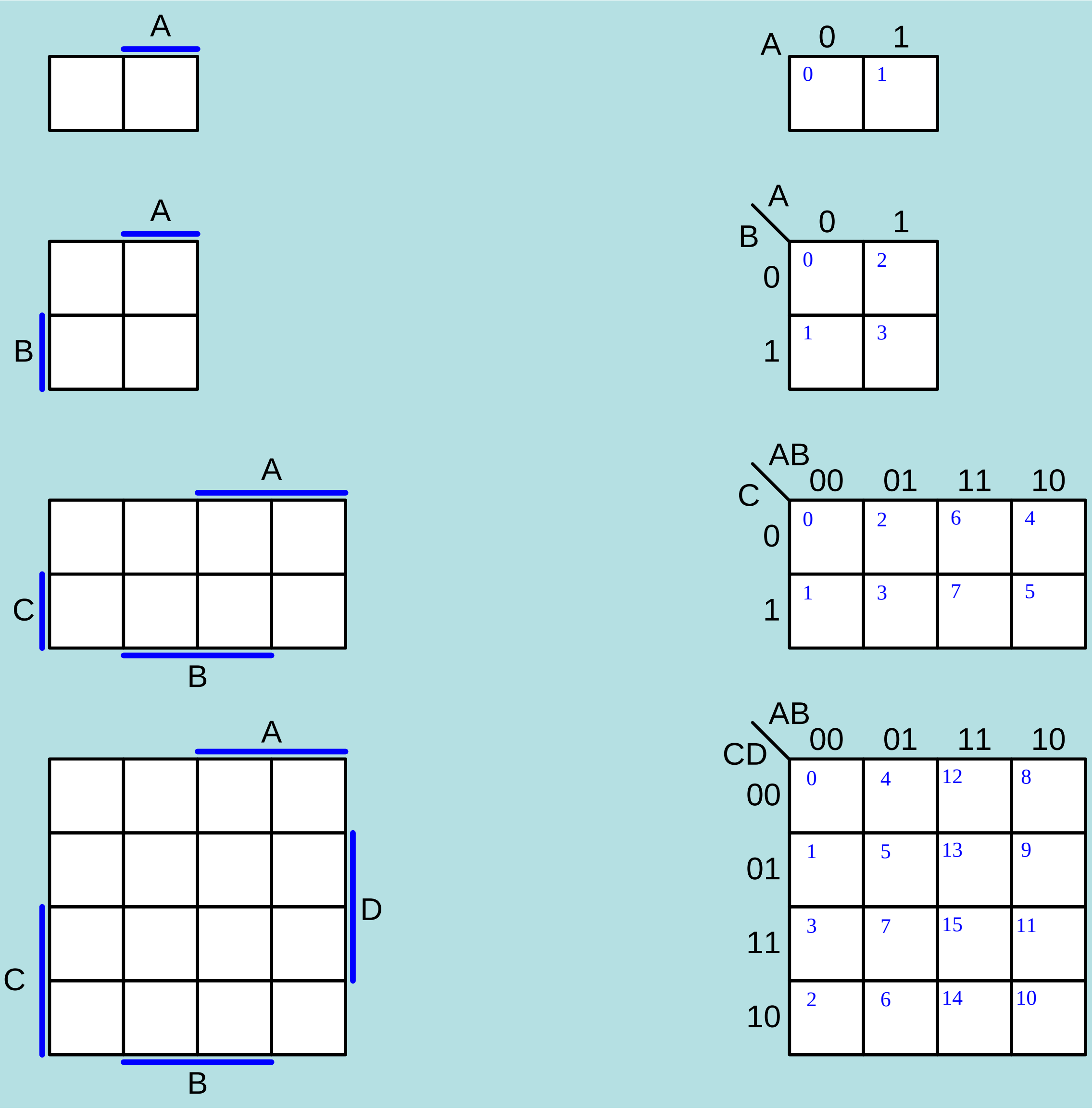
- Example 8:
  - *For the given Venn Diagram, find the logic function :*



F (A, B, C) = ?

V. K-Diagrams

- Representations of Boolean Func. with K-Diagrams (*Karnaugh Diag*):



V. K-Diagrams

- Example 9:
  - *Draw the K-Diagrams for the following functions:*

$$a)\quad F_1 = A \cdot B + C$$

$$b)\quad F_2 = A \cdot \overline{B} + \overline{A} \cdot B$$

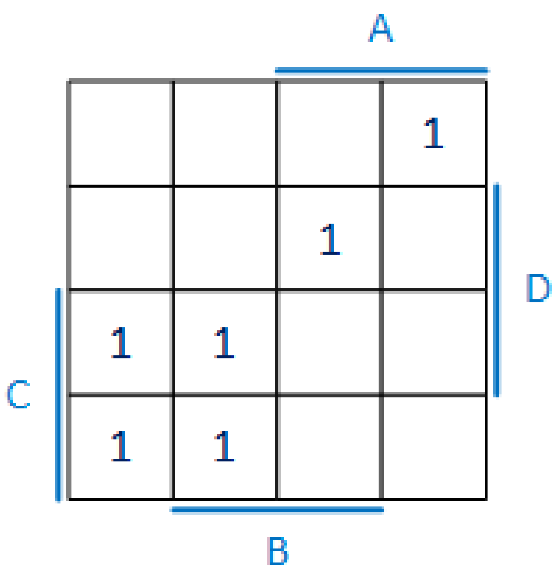
$$c)\quad F_3 = A \cdot B + C \cdot (\overline{A} + B)$$

V. K-Diagrams

- Example 10:
  - *For the given K-Diagram, find:*

    a) The logic function and **F = ?**

    b) The Invers function $\overline{\mathbf{F}} = ?$



- Questions?!

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