AAB University

Faculty of Computer Sciences

Introduction to Digital Technologies and Circuits

Week 3:

Binary Arithmetic and Codes

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Last Time

- *Number Systems*
  - *Not weighted Number Systems*
  - *Weighted Number Systems*
- *Conversions between number systems*
- *Direct conversions between number systems*

Today

- *Binary Arithmetic*

&

Codes

Binary Arithmetic

- Binary Arithmetic:
  - Addition
  - Subtraction
  - Multiply
  - Divide

- Example:

$$X_2 = \frac{F3A_{16} - 2112_8}{130_4}$$

Binary Arithmetic

- Addition:

$$0+0=0$$
$$0+1=1$$
$$1+0=1$$
$$1+1=0 \quad \textit{carry } 1$$

- Example: $(13)_{10} + (11)_{10} = ?$

$$\begin{array}{r} 1111 \\ 13_{10} = \quad 1101 \\ 11_{10} = \quad 1011 \\ \hline 11000 = 24_{10} \end{array}$$

Binary Arithmetic

- Subtraction:

$$0-0=0$$
$$0-1=1 \quad \textit{borrow } 1$$
$$1-0=1$$
$$1-1=0$$

- Example: $(29)_{10} - (19)_{10} = ?$

$$\begin{array}{r} {\scriptstyle 1} \\ 11101 \\ -\ 10011 \\ \hline 1010 \end{array}$$

Binary Arithmetic

- Multiply:

$$0*0=0$$
$$0*1=0$$
$$1*0=0$$
$$1*1=1$$

- Example: $(13)_{10} * (11)_{10} = ?$

$$\begin{array}{r} 1101 \\ 1011 \\ \hline 1101 \\ 1101 \\ 0000 \\ 1101 \\ \hline 10001111 = 143_{10} \end{array}$$

Binary Arithmetic

- Divide:

$$0:0=?$$
$$0:1=0$$
$$1:0=?$$
$$1:1=1$$

- Example: $(145)_{10} : (11)_{10} = ?$

```
      1101
1011 | 10010001
       1011
        1110
        1011
          1101
          1011
            10
```

Octal Arithmetic ☺

- Example:

```
   3 5
 + 4 4
-------
 1 0 1
```

```
 1 0 2
 - 2 7
-------
   5 3
```

```
     2 3
   *   5 2
-----------
     4 6
 + 1 3 7
-----------
   1 4 3 6
```

1 4 4 : 3 1 = 4

Hexadecimal Arithmetic ☺

- Example:

```
   C E D E
+    D E F
---------
   D C C D
```

```
   B O B O
-  A 1 B 2
---------
     E F E
```

```
     B C
*      A
---------
 7   5  8
```

```
B C : A = 1 2 . C C
A
---
1 C
1 4
---
  8 0
  7 8
  ---
    8
```

Binary Arithmetic

- Example:

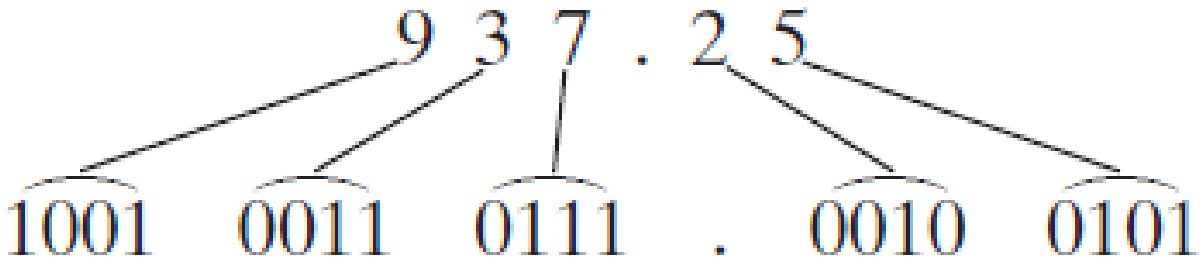
$$X_2 = \frac{F3A_{16} - 2112_8}{130_4}$$

?

Binary Codes

- Although computers work internally with binary numbers, the input-output equipment generally uses decimal numbers. Because most logic circuits only accept two-valued signals, the decimal numbers must be coded in terms of binary signals.
  - *In the simplest form of binary code, each decimal digit is replaced by its binary equivalent.*

- Example: (**937.25**) = **?**



Binary Codes

- Binary codes are codes which are represented in binary system with modification from the original ones.

- Weighted Binary Codes
  - Weighted binary codes are those which obey the positional weighting principles, each position of the number represents a specific weight. The binary counting sequence is an example.

- Non Weighted Codes
  - Non weighted codes are codes that are not positionally weighted. That is, each position within the binary number is not assigned a fixed value.

Weighted Binary Codes

- **8421 Code/BCD Code**
  - The BCD (Binary Coded Decimal) is a straight assignment of the binary equivalent. It is possible to assign weights to the binary bits according to their positions. The weights in the BCD code are 8,4,2,1.

- Example:
  - The bit assignment 1001, can be seen by its weights to represent the decimal 9 because:

$$1x8+0x4+0x2+1x1 = 9$$

Weighted Binary Codes

- Binary Codes for Decimal Digits:

Decimal	8421	2421	5211	Excess-3
0	0000	0000	0000	0011
1	0001	0001	0001	0100
2	0010	0010	0011	0101
3	0011	0011	0101	0110
4	0100	0100	0111	0111
5	0101	1011	1000	1000
6	0110	1100	1010	1001
7	0111	1101	1100	1010
8	1000	1110	1110	1011
9	1001	1111	1111	1100

Weighted Binary Codes

- **2421 Code**
  - This is a weighted code, its weights are 2, 4, 2 and 1. A decimal number is represented in 4-bit form and the total four bits weight is 2 + 4 + 2 + 1 = 9. Hence the 2421 code represents the decimal numbers from 0 to 9.

- **5211 Code**
  - This is a weighted code, its weights are 5, 2, 1 and 1. A decimal number is represented in 4-bit form and the total four bits weight is 5 + 2 + 1 + 1 = 9. Hence the 5211 code represents the decimal numbers from 0 to 9.

Non Weighted Binary Codes

- **Excess-3 Code**
  - Excess-3 is a **non weighted** code used to express decimal numbers. The code derives its name from the fact that each binary code is the corresponding 8421 code plus 0011(3).

- Example:

  1000 of 8421 = 1011 in Excess-3

Non Weighted Binary Codes

- Construct a **5-2-2-1** weighted code for decimal digits.
  - What numbers does **1110 0110** represent in this code?

Non Weighted Binary Codes

- Construct a **5-4-1-1** weighted code for decimal digits?!!

Non Weighted Binary Codes

- Write the decimal number **3 5 6 1 7 6** using the following codes:

a) **NBCD**

b) **6-3-1-1**

c) **4-3-2-1**

d) **8-4-(-2)-(-1)**

e) **5-2-1-1**

Non Weighted Binary Codes

- What number does **0001 1001 0111** represent in the code **6-2-2-1 ?**

Non Weighted Binary Codes

- **Gray Code**
  - The gray code belongs to a class of codes called minimum change codes, in which only one bit in the code changes when moving from one code to the next. The Gray code is non-weighted code, as the position of bit does not contain any weight. The gray code is a reflective digital code which has the special property that any two subsequent numbers codes differ by only one bit. This is also called a unit-distance code. In digital Gray code has got a special place.

Non Weighted Binary Codes

- Gray Code

Decimal Number	Binary Code	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Other Codes

- Sequential Codes
- Cyclic Codes
- Optimal Codes
- Error Detecting and Correction Codes
- Alphanumeric Codes

Alphanumeric Codes

- Many applications of computers require the processing of data which contains numbers, letters, and other symbols such as punctuation marks.
  - *In order to transmit such alpha numeric data to or from a computer or store it internally in a computer, each symbol must be represented by a binary code.*

- **ASCII** code
  - *American Standard Code for Information Interchange*

- **UNICODE**
  - *More modern code that can represent 65536 characters/symbols*
  - *useful for other languages such as Arabic, Chinese…*

ASCII code

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	|
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

Alphanumeric Codes

- Give the ASCII code for your name **?!**

- Questions?!

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