



A A B University

Faculty of Computer Sciences

Introduction to Digital Technologies and Circuits

Week 6, 7:

Logic Design of Combinational Circuits

Asst. Prof. Dr. **Mentor Hamiti**
mentor.hamiti@universitetiaab.com



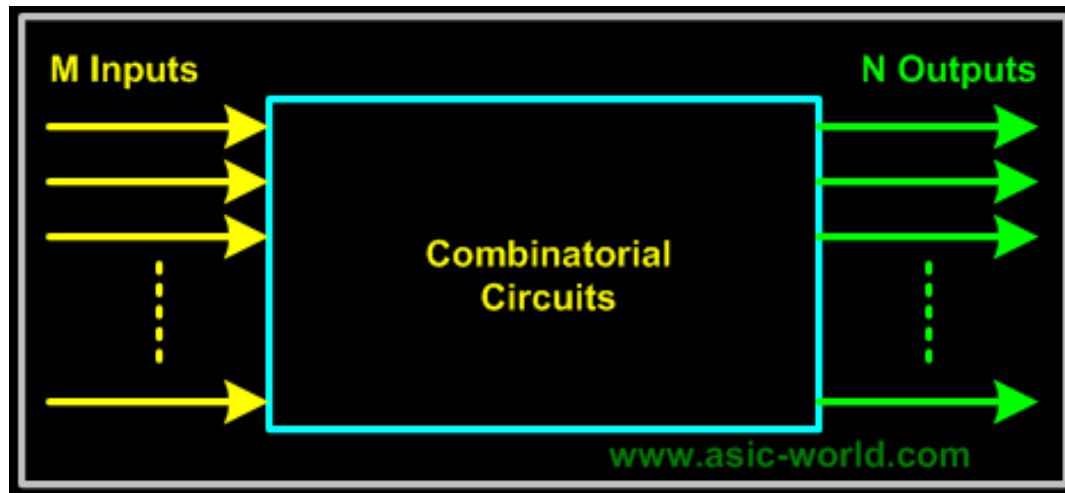
- Minimization of Logic Functions
- Combinational Logic Circuits
- Logic Gates
 - Basic Logic Gates
 - Universal Logic Gates
 - Special (exclusive) Logic Gates



- **Combinational Logic Circuits**
 - Decoders
 - Encoders
 - Multiplexers
 - De-multiplexers
 - Other Combinational Circuits



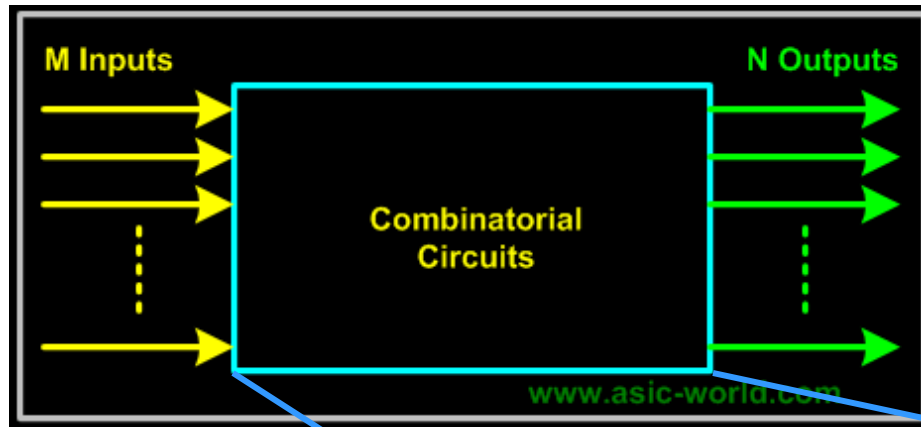
- Combinational Circuits are circuits which can be considered to have the following generic structure.



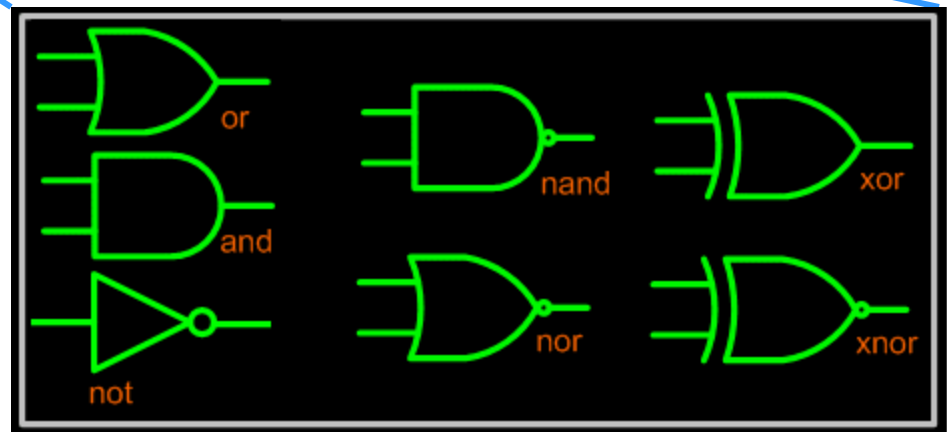
- Whenever the same set of inputs is fed in to a combinational circuit, the same outputs will be generated. Such circuits are said to be stateless.



- All Combinational Circuits are composed by simple combinational logic elements - “**Logic Gates**”.



- Logic Gates:
 - Basic Logic Gates
 - Universal Logic Gates
 - Exclusive Logic Gates

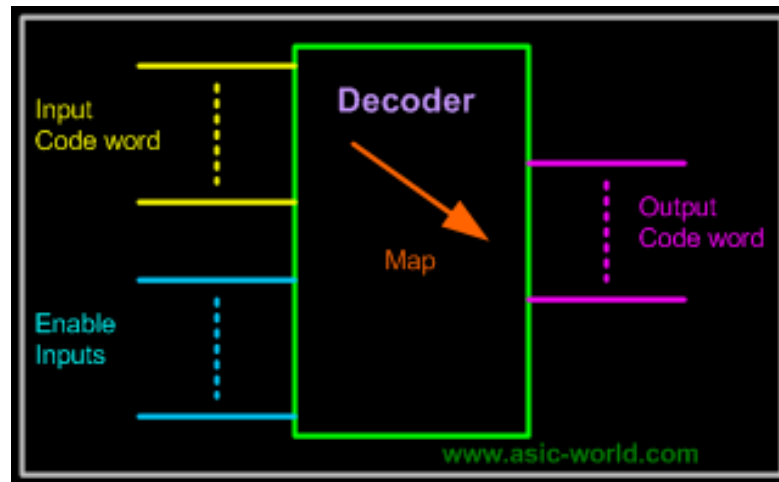




- **Combinational Circuits:**
 - Decoders
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- A decoder is a multiple-input, multiple-output logic circuit that converts coded inputs into coded outputs, where the input and output codes are different.
- Decoding is necessary in applications such as data multiplexing, 7 segment display and memory address decoding.



- Enable inputs must be on for the decoder to function, otherwise its outputs assume a single "disabled" output code word.



- Binary **n-to- 2^n** Decoder:



- A binary decoder has n inputs and 2^n outputs.
- Only one output is active** at any one time, corresponding to the input value.

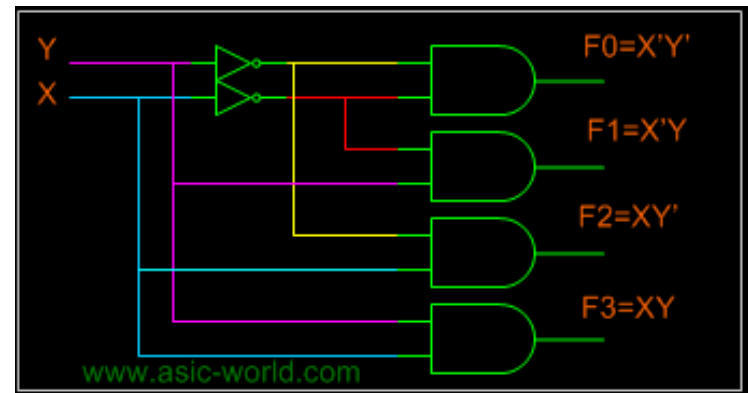
- Binary Decoder **2-to-4**:



Truth Table:

X	Y	F ₀	F ₁	F ₂	F ₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

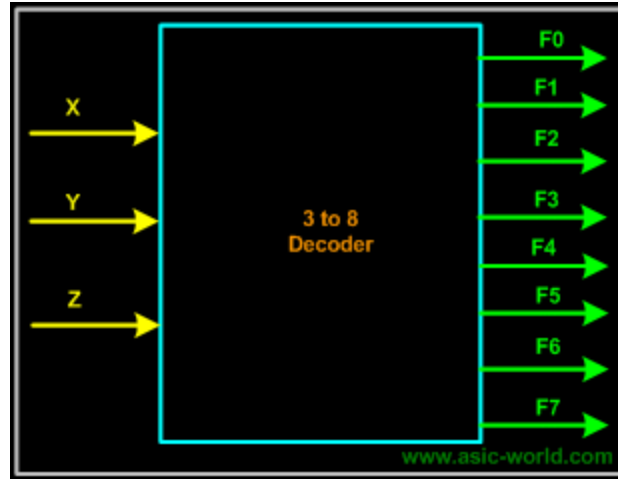
Logic Circuit:



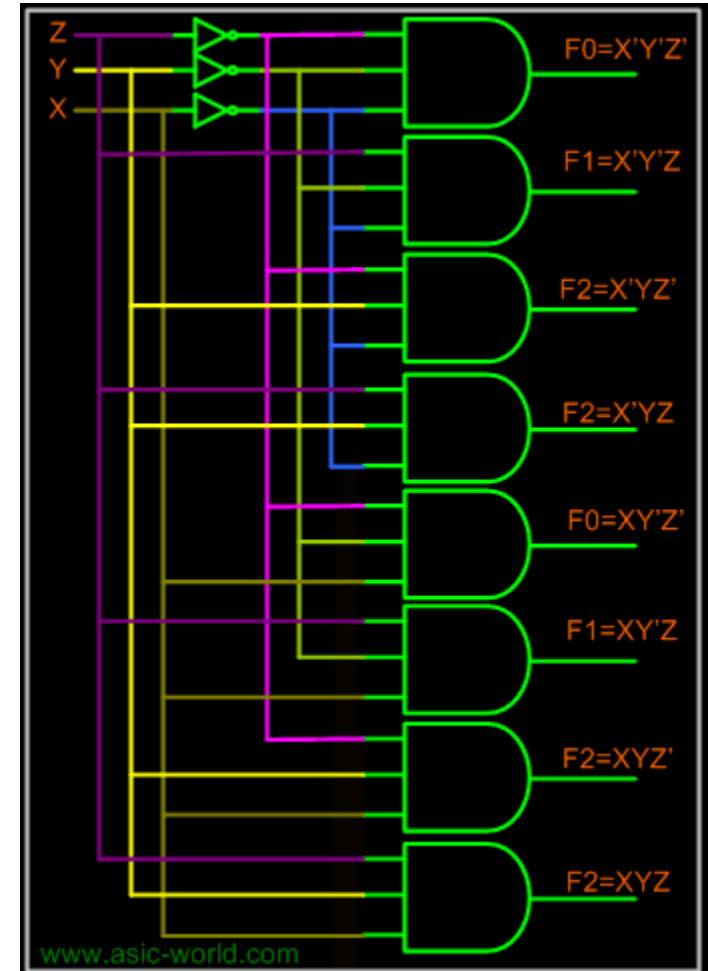
- Each output is a 2-variable minterm ($X'Y'$, $X'Y$, XY' , XY)



- Binary Decoder
3-to-2³:



X	Y	Z	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

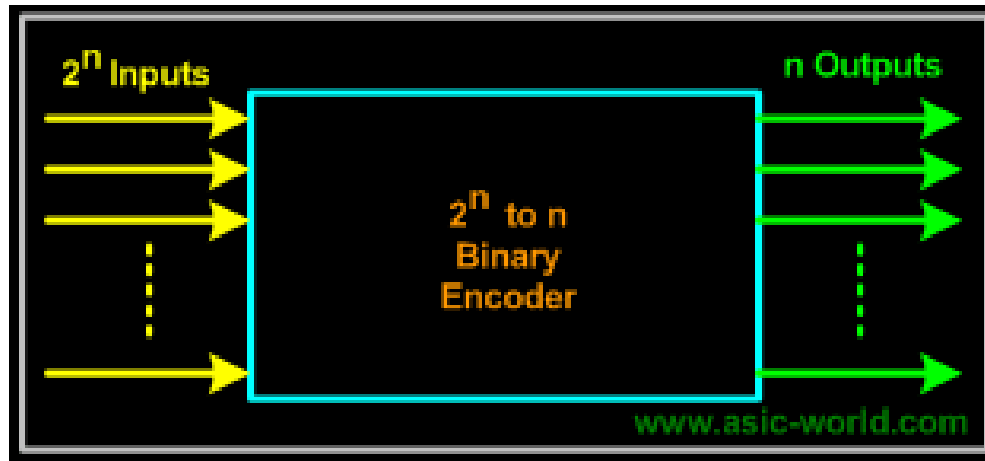




- Combinational Circuits:
 - Decoders
 - Encoders
 - Multiplexers
 - De-multiplexers
 - Other Combinational Circuits



- An encoder is a combinational circuit that performs the inverse operation of a decoder.



- The simplest encoder is a **2ⁿ-to-n** binary encoder, where it has only one of 2^n inputs = 1 and the output is the n-bit binary number corresponding to the active input.



- **Octal-to-Binary Encoder:**

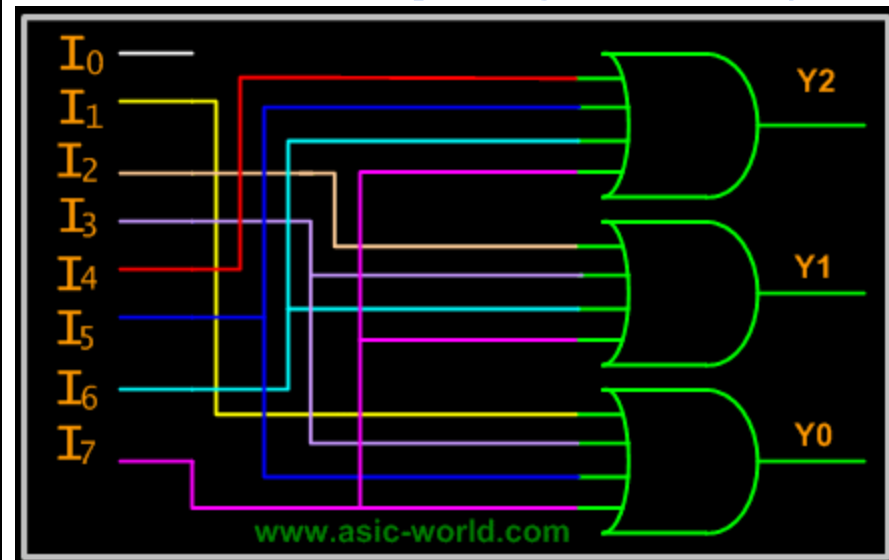
- Octal-to-Binary take 8 inputs and provides 3 outputs, thus doing the opposite of what the 3-to-8 decoder does. At any one time, only one input line has a value of 1.

- The logic expressions of the outputs are: $Y_0 = I_1 + I_3 + I_5 + I_7$

$$Y_1 = I_2 + I_3 + I_6 + I_7$$

$$Y_2 = I_4 + I_5 + I_6 + I_7$$

I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7	Y_2	Y_1	Y_0
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1





- **Decimal-to-Binary** Encoder

- Decimal-to-Binary take 10 inputs and provides 4 outputs, thus doing the opposite of what the 4-to-10 decoder does. At any one time, only one input line has a value of 1.

I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	Y_3	Y_2	Y_1	Y_0
1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	0	0	0	0	1	1	0
0	0	0	0	0	0	0	1	0	0	0	1	1	1
0	0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	1	0	0	1

Derived functions:

$$Y_3 = I_8 + I_9$$

$$Y_2 = I_4 + I_5 + I_6 + I_7$$

$$Y_1 = I_2 + I_3 + I_6 + I_7$$

$$Y_0 = I_1 + I_3 + I_5 + I_7 + I_9$$



- If more than two inputs are active simultaneously at the Encoder circuits, the output is unpredictable or rather it is not what we expect it to be.
- This ambiguity is resolved if priority is established so that only one input is encoded, no matter how many inputs are active at a given point of time.
- The priority encoder includes a priority function. The operation of the priority encoder is such that if two or more inputs are active at the same time, the input having the highest priority will take precedence.



- Priority Encoder **4 to 3**:
- A 4 to 3 encoder consists of four inputs and three outputs. The input D₃ has the highest priority, D₂ has next highest priority, D₀ has the lowest priority. This means output Y₂ and Y₁ are 0 only when none of the inputs D₁, D₂, D₃ are high and only D₀ is high.

Truth Table:

D ₃	D ₂	D ₁	D ₀	Y ₂	Y ₁	Y ₀
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	x	0	1	0
0	1	x	x	0	1	1
1	x	x	x	1	0	0

Derived functions:

K-diagrams ...

$$Y_2 = \dots = D_3$$

$$Y_1 = \dots = D_1 D'_3 + D_2 D'_3$$

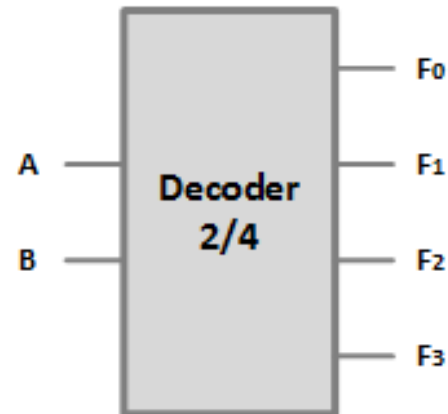
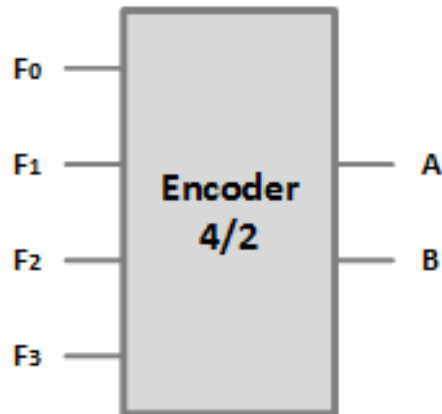
$$Y_0 = \dots = D_0 D'_1 D'_3 + D_2 D'_3$$

Circuit ...



■ Example 1:

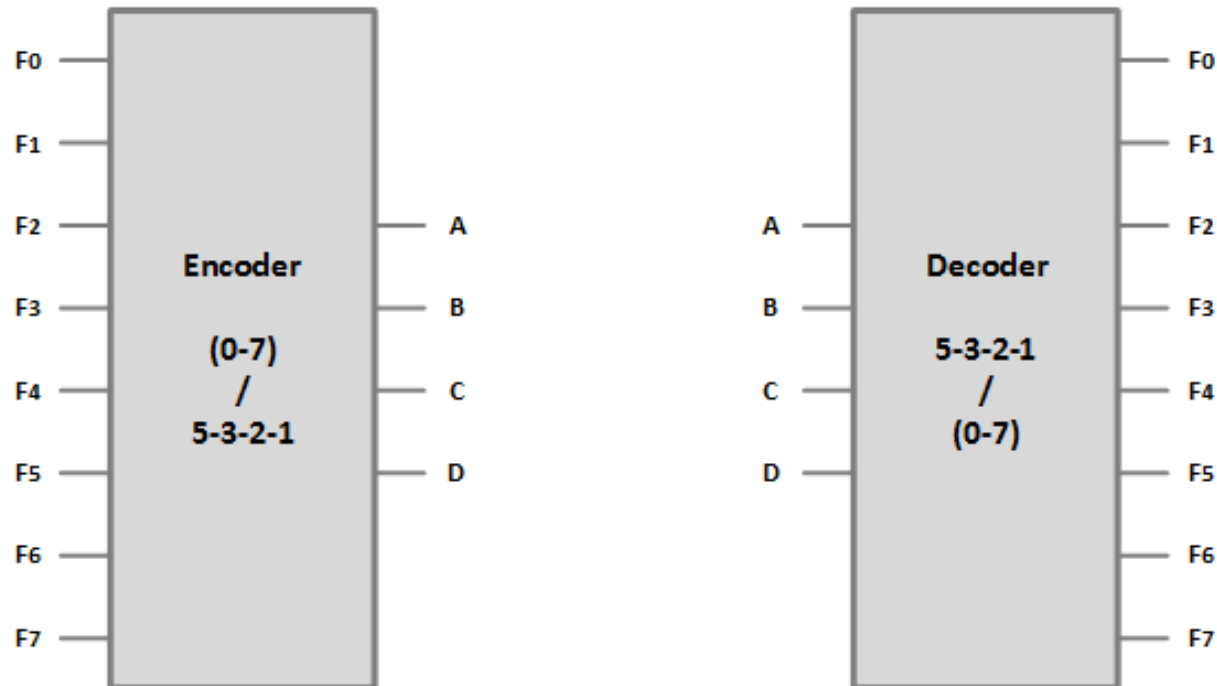
- Design **Encoder (4/2)** and **Decoder (2/4)**, with the use of Basic Logic Gates!





■ Example 2:

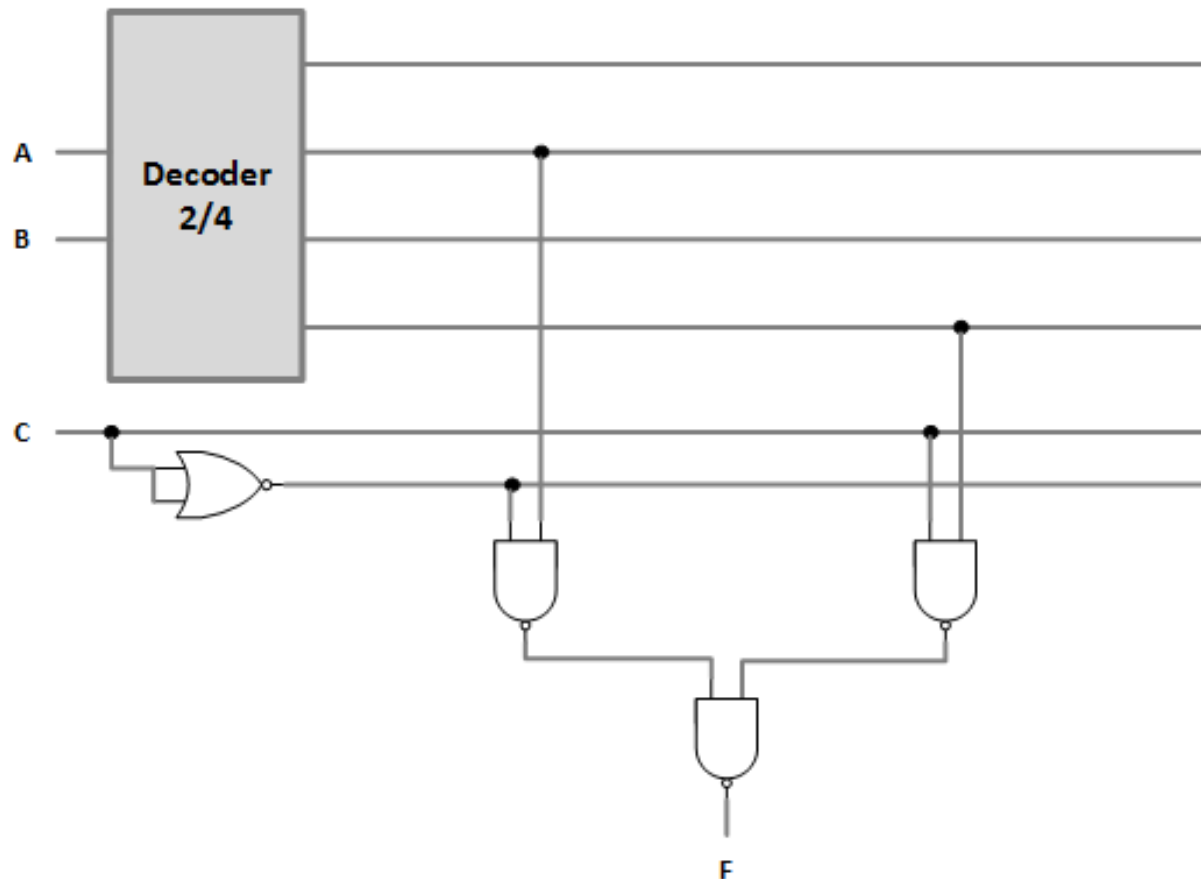
- Construct encoder and decoder for coding and decoding **octal numbers** into and from the weighted code **5-3-2-1**.





■ Example 3:

- Find the Logic Function for the given Combinational Logic Circuit and after that design the appropriate Timing Diagram!





■ Example 4:

- If the Logic Function is given by the following Timing Diagram:



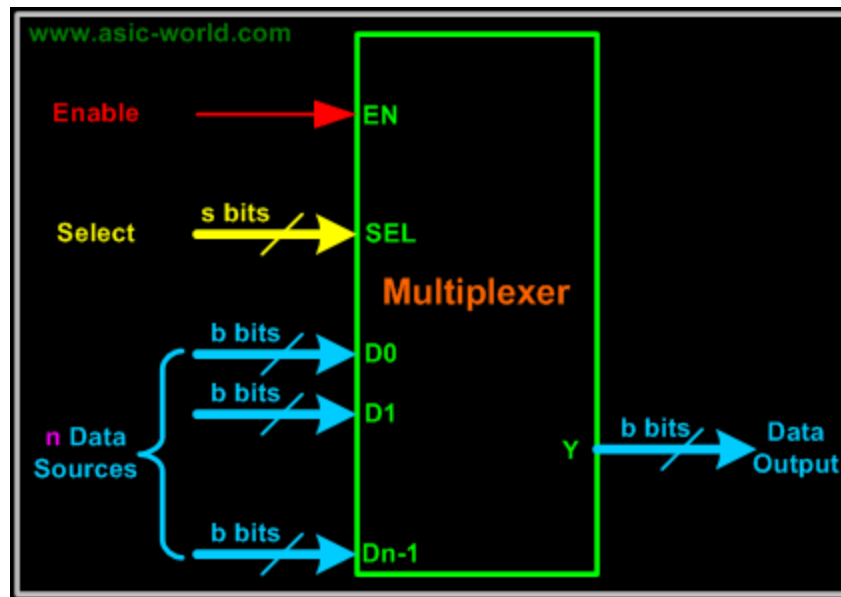
Design the Combinational Logic Circuit, with the use of two **Decoders (2/4)** and **Basic Logic Gates!**



- Combinational Circuits:
 - Decoders
 - Encoders
 - Multiplexers
 - De-multiplexers
 - Other Combinational Circuits



- A multiplexer (**MUX**) is a digital switch which connects data from one of **n** sources to the output.
- A number of select inputs determine which data source is connected to the output.
- The block diagram of **MUX** with **n** data sources of **b** bits wide and **s** bits wide select line:



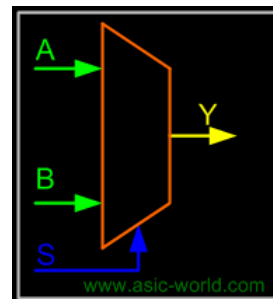


- Mechanical Equivalent of a Multiplexer
- The operation of a multiplexer can be better explained using a mechanical switch as shown:



- This rotary switch can touch any of the inputs, which is connected to the output. As it is seen at any given point of time only one input gets transferred to output.

- MUX **2x1**:



S	Y
0	A
1	B



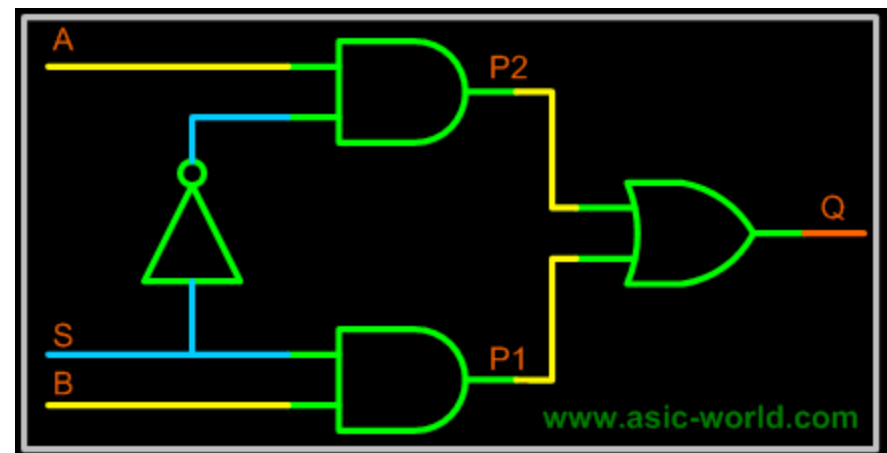
- Mux **2:1**

B	A	S	Q
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Inputs: A, B, S

Output: $Q = A \cdot \bar{S} + B \cdot S$

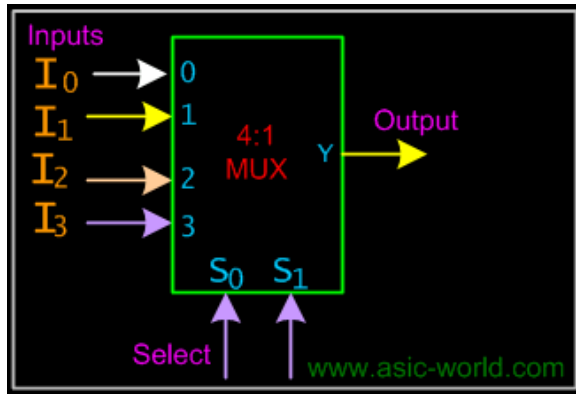
Circuit:





■ MUX 4:1

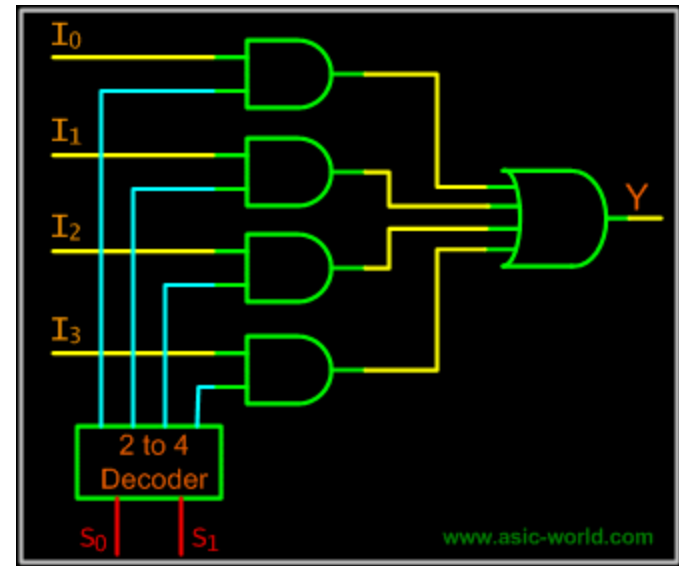
Symbol:



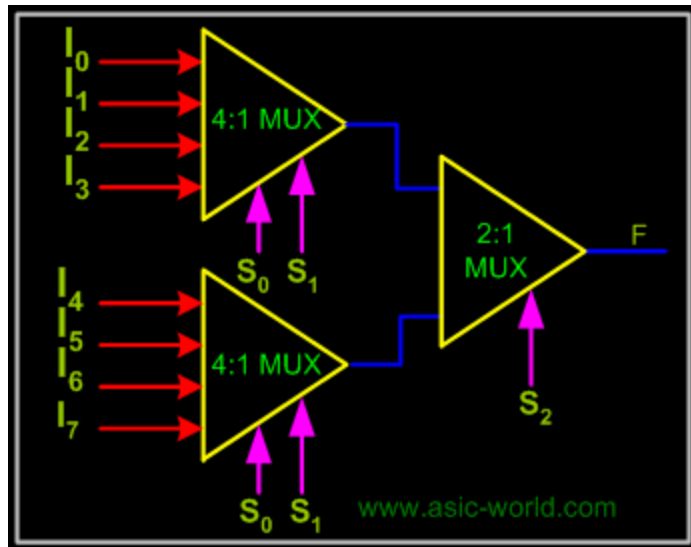
Truth Table

S1	S0	Y
0	0	I ₀
0	1	I ₁
1	0	I ₂
1	1	I ₃

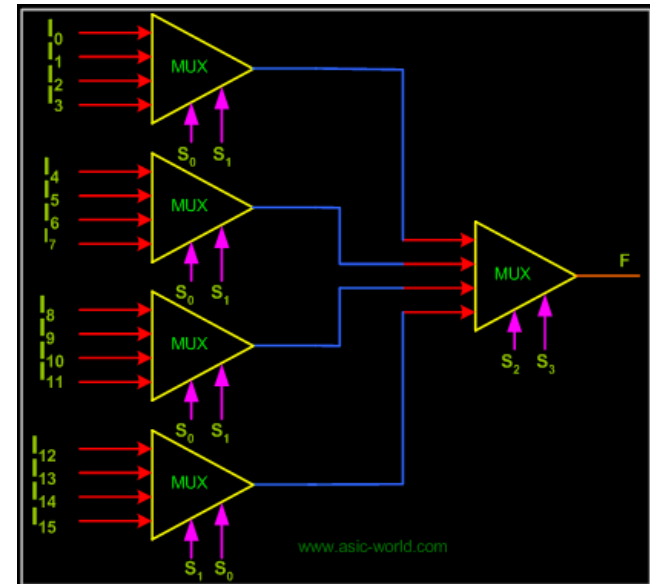
Circuit:



- Larger Multiplexers
 - can be constructed from smaller ones.
- Example:
 - An 8-to-1 multiplexer can be constructed from smaller multiplexers.

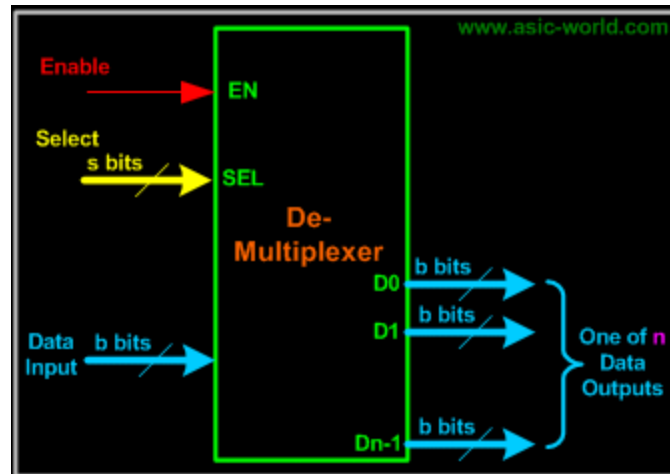


16-to-1 mux from **4:1** mux



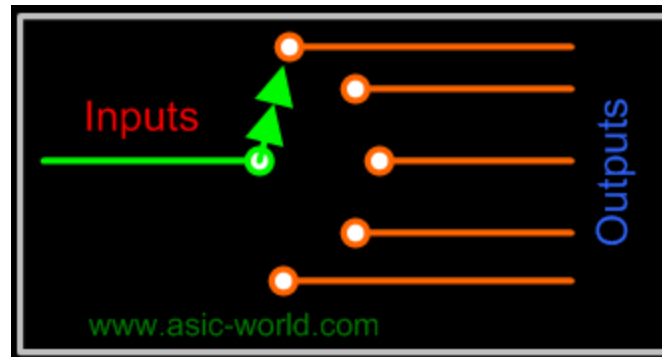


- They are digital switches which connect data from **one** input source to **one** of **n** outputs. Usually implemented by using **n-to- 2^n** binary decoders where the decoder enable line is used for data input of the de-multiplexer.
- De-multiplexer block diagram:
 - which has got **s**-bits-wide select input, one **b**-bits-wide data input and **n** b-bits-wide outputs.





- Mechanical Equivalent of a De-multiplexer
- The operation of a de-multiplexer can be better explained using a mechanical switch as shown below:



- This rotary switch can touch any of the outputs, which is connected to the input, and at any given point of time only one output gets connected to input.

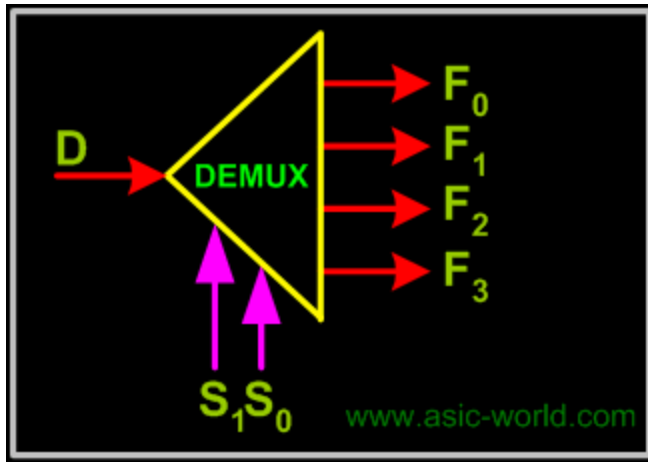


- **Combinational Circuits:**
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- De-multiplexer **1-to-4**:

Symbol:



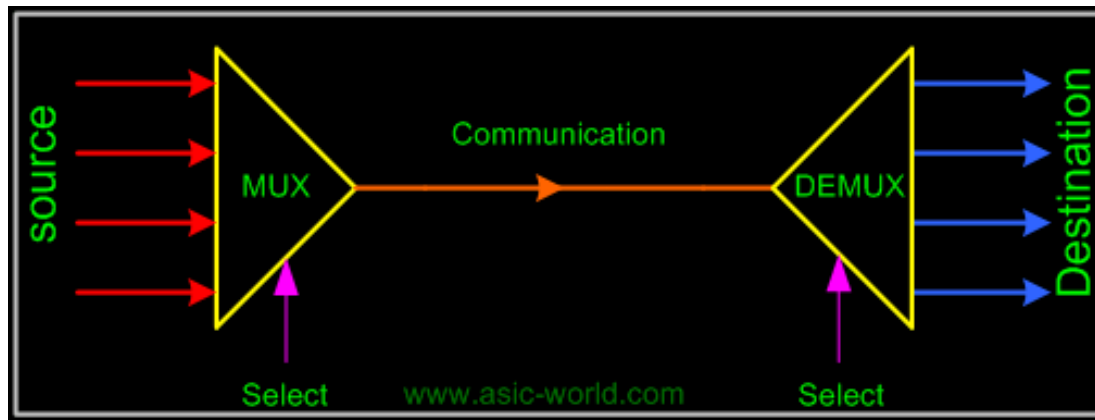
Truth table:

S_1	S_0	F_0	F_1	F_2	F_3
0	0	D	0	0	0
0	1	0	D	0	0
1	0	0	0	D	0
1	1	0	0	0	D

Mux-Demux Application Example



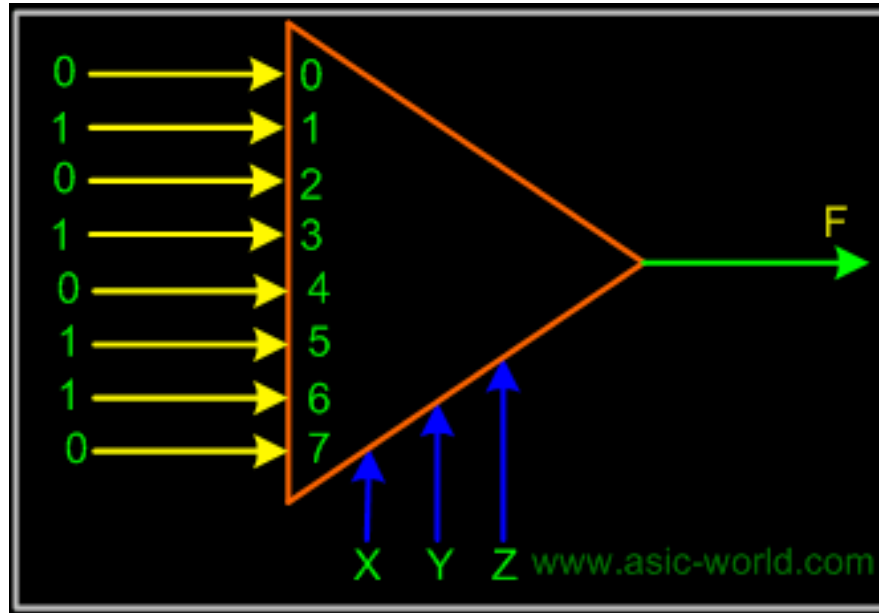
- This enables sharing a single communication line among a number of devices. At any time, only one source and one destination can use the communication line.





■ Example 1:

- Find the output of the following **8-to-1 multiplexer**?

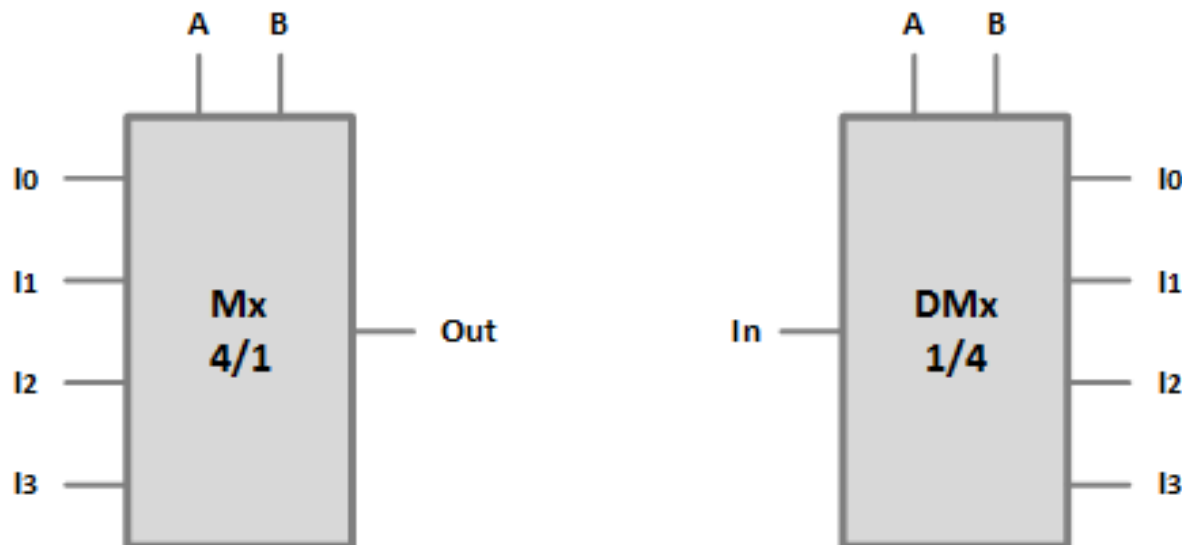


$$F(X,Y,Z) = m^1(1,3,5,6)$$



■ Example 2:

- Design **Multiplexer (4/1)** and **De-multiplexer (1/4)**, with the use of Basic Logic Gates!



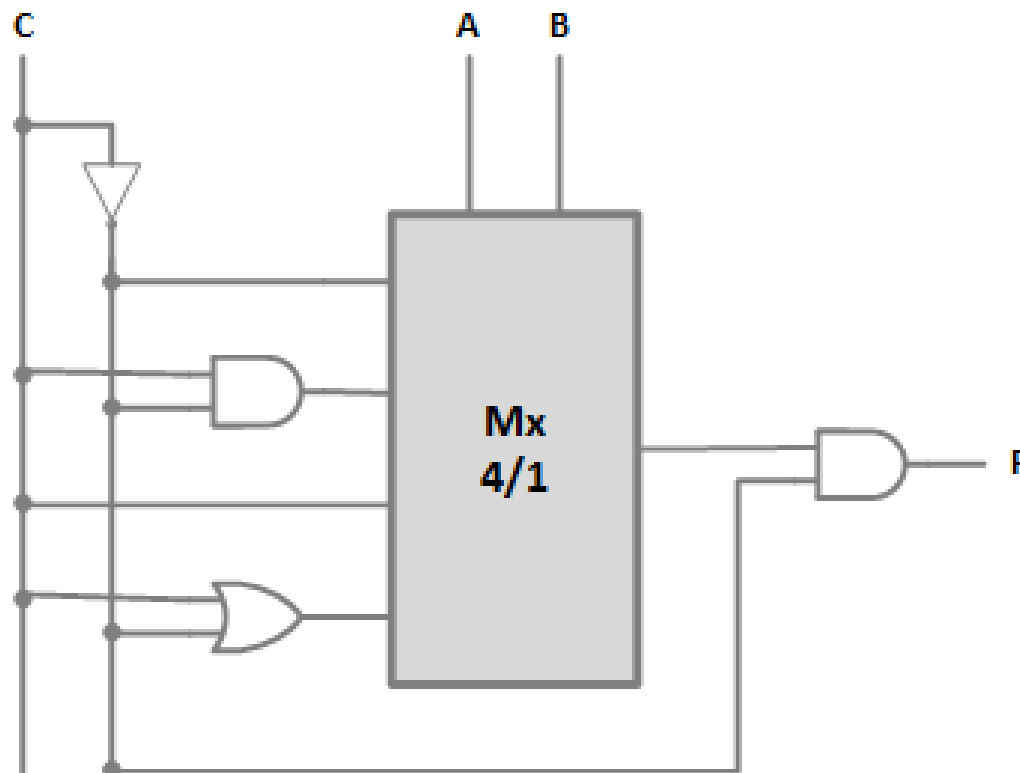


- Example 3:
 - Show how **two 4-to-1** and **one 2-to-1 multiplexers** could be connected to form an **8-to-1 MUX** with three control inputs?!



■ Example 4:

- Find the Logic Function for the given Combinational Logic Circuit and after that design the appropriate Timing Diagram and the equivalent circuit using Basic Logic Gates!

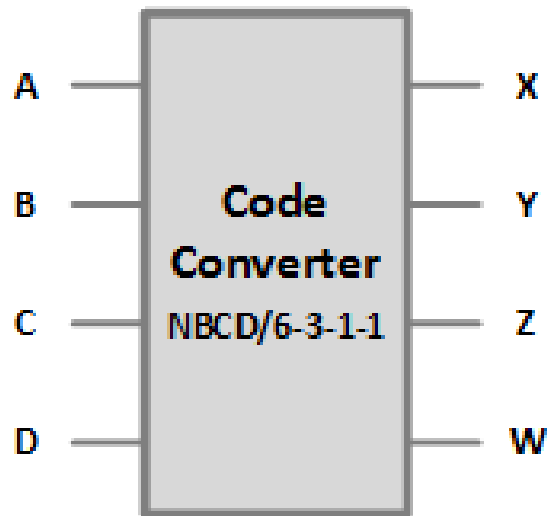




- **Combinational Circuits:**
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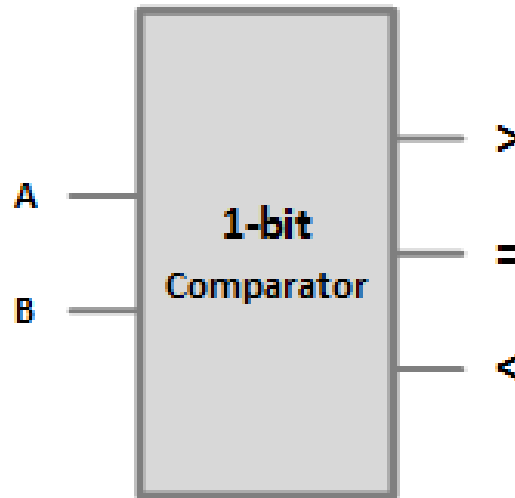


- Example 5:
 - Design the Code converter **NBCD/6-3-1-1**





- Example 6:
 - Design the **1-bit comparator**

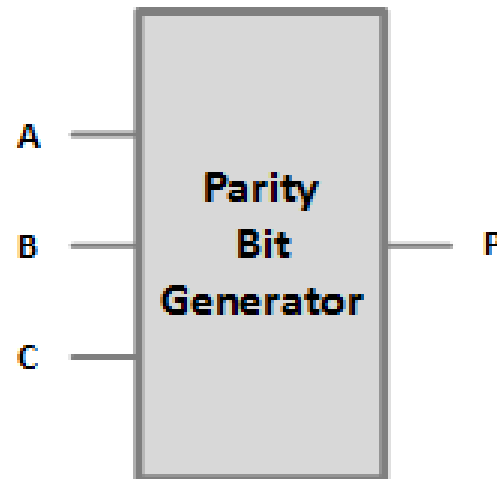




■ Example 7:

- Design the **Parity Bit Generator** for three digits words!

A	B	C	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1





- Questions?!

